The factorization in exclusive B decays: a critical look

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I review the theoretical ideas and concepts along the line of factorization in the exclusive B decays. In order to understand the naive factorization, the effective field theories and the perturbative method of QCD are introduced and developed. We focus our discussions on the large energy effective theory, the QCD factorization approach and the soft-collinear effective theory.

1 Introduction

The exploration of CP violation and determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements motivate extensive interests of B meson decay. From another point of view, B decays provide a good place to study the fruitful dynamics of QCD. Up to now, we have not a truly successful method to calculate the non-perturbative QCD and the mechanism of quark confinement is still unknown. The study of exclusive B decays is usually difficult because of the complicate QCD dynamics. However, the experiments from the Belle and Babar collaborations have accumulated and will continue to accumulate a large amount of data of B decays. The large theoretical uncertainties cannot compete with the more precise experimental data. We come to one stage that experiment goes ahead of theory. The theorists in B physics have to meet great challenge from the experiment.

The problem of exclusive B decays lies in a very large number of degrees of freedom. The experiment observes the hadron states such as B meson and pion, kaon etc. In the QCD Lagrangian, only quark and gluon degrees of freedom appear. We don't know accurately how the hadron are formed by quarks and gluons. From the energy scale standpoint, the B decays usually contain many scales: the weak interaction scale m_W, the b quark mass m_b, the QCD scale Λ_{QCD} and possible intermediate scales due to the soft spectator quark in B meson. The momenta of quarks or gluons are not restricted. They can be highly virtual, very soft or highly energetic but collinear to the fast moving pion. The fact that we have to treat all the degrees of freedom in one process if we think QCD is the correct theory of strong interaction leads to great theoretical complications.

One method to treat the multi-scales problem is factorization. The factorization is a key ingredient of perturbative QCD (pQCD) [1, 2]. Its basic idea is to separate the short-distant dynamics from the long-distance physics. It has been widely used in the hard QCD processes where the large momentum transfer $Q \gg \Lambda_{QCD}$ is involved. Another method is the effective field theory. It is a useful toll to study the process with several separate scales. The heavy quark effective theory (HQET) [3] is a low energy effective theory. It allows model-independent predictions in some cases of the heavy meson system, such as $B \to D$ form factor at zero recoil. The developments of the two methods are nearly independent although some ideas in them are related. As we will show that these two lines of thought converge in the study of exclusive B decays,.

The factorization had been introduced in exclusive B decays for a long time. The old form which we call the naive factorization approach is to divide a hadronic matrix element into the multiplication of a form factor and hadron decay constant. This idea influences B physics for more than 30 years. Much efforts were done to interpret and generalize it. Now, the idea of factorization has been developed as a central idea of B physics. In this talk, we will discuss the theoretical struggle of studying the exclusive B decays along the line of factorization. The success and limitations of each theoretical approach will be analyzed. We are focus on the conceptual developments from the naive factorization approach to the QCD factorization approach. We will show how the effective field theory enters into B physics and modifies our view.

2 The naive factorization approach and the large energy effective theory

The first thing to do in B decays is to integrate out the heavy degrees of freedom of W, Z bosons and top quark in the standard model. The method is to construct an effective theory where the above heavy particles do not appear. The theoretical technic is mature now. It uses the operator product expansion (OPE) and renormalization group equation (RGE). For non-

leptonic B decays, the relevant effective weak Hamiltonian is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\text{CKM}} C_i(\mu) Q_i. \tag{1}$$

where G_F is the Fermi constant and Q_i are current-current operators. The scale μ is chosen of order of m_b. The amplitude of $B \to M_1 M_2$ decay is

$$A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\text{CKM}}$$
$$\times C_i(\mu)\langle M_1 M_2 | Q_i | B \rangle(\mu). \tag{2}$$

where $\langle M_1 M_2 | Q_i | B \rangle$ are hadronic matrix elements. The remained work is to calculate the hadronic matrix element.

2.1 The naive factorization approach

The introduction of factorization to simplify the the hadronic matrix element may be firstly given in [4] up to knowledge of the author. I cannot trace out this history but refer to [5] as our start of discussion. Bauer, Stech and Wirbel consider the non-leptonic two meson decays where the final mesons are energetic. They made assumptions that only the asymptotic part of the hadron field is effective and the current are proportional the hadron field. All the initial state interaction and final state interactions are neglected. Based on the above assumptions, one hadron and its associated current are separated out. The hadronic matrix element is factorized into a multiplication of decay constant and the form factor which represented by the matrix element of the other current. Take $\bar{B}^0 \to \pi^+\pi^-$ decay as an example,

$$\langle \pi^+\pi^- |(\bar{u}b)_{\text{V}-\text{A}}(\bar{d}u)_{\text{V}-\text{A}}|\bar{B}^0\rangle =$$
$$\langle \pi^- |(\bar{d}u)_{\text{V}-\text{A}}|0\rangle\langle \pi^+ |(\bar{u}b)_{\text{V}-\text{A}}|\bar{B}^0\rangle. \tag{3}$$

The idea of the above factorization is simple but it has a deep influence. The application of the above naive factorization approach into the non-leptonic two body B decays is successful in early days of B physics when the experimental data are rare. For a long time, this approach is nearly the only method to give a theoretical prediction of exclusive non-leptonic B decays although the accuracy is at the qualitative level for many processes.

The factorization approach plays a similar role as the Feynman's parton model in DIS, we can call this naive approach as the parton model in B physics. One might expect that the factorization is a limit case of a more general theory. The understanding of the factorization from field theory of QCD is a long way. The first step

comes from Bjorken's intuitive space-time picture [6]. It is Bjorken who proposed the famous scaling in DIS which lead to the rise of QCD. For $\bar{B}^0 \to \pi^+\pi^-$ decay, the quark level decay is $b \to u + \bar{u}d$. In order to form the final energetic hadron, the quark pair $\bar{u}d$ has to choose a nearly collinear configuration. Because the pions move fast, the formation time of π^- will be long because of the relativistic time-dilation. The dilation ratio is $m_b/\Lambda_{\text{QCD}} \approx 20$. That means the hadronization occurs 20 fm away from the remained system. Before the hadronization, the $\bar{u}d$ quark pair produced from the pointlike, color-singlet weak interaction is a a small color dipole. The small color dipole has little interaction with the other quarks. The above consideration is usually called "color transparency argument". From the above argument, one may guess that the factorization approach is the leading order contribution of heavy quark limit where $m_b \gg \Lambda_{\text{QCD}}$ and the non-factorizable corrections come from the interactions of small color dipole with the remained quarks at short distance.

What is color transparency? It is a concept outside of B physics. According to the discussions in [7], the color transparency is a phenomenon of pQCD [2]. It says that a small color-singlet object can pass freely through nucleon target as if the target is transparent. The large target acts as a filter which removes the large transverse separation component of the hadron. The $\bar{B}^0 \to D^+\pi^-$ decay provides a similar environment. $B \to D$ transition is at long distance. The energetic π^- selects the $\bar{u}d$ quark pair at small transverse separations. The long distance processes caused by emitting or absorbing soft gluons have destructive effects and cancel.

In [8], Politzer and Wise apply the pQCD method into the exclusive processes in $\bar{B}^0 \to D^+\pi^-$ decay. They proposed a factorization formula that the hadronic matrix element can be written as the product of a matrix element of B and D mesons in HQET and a convolution by a hard scattering amplitude T and the pion distribution amplitude $\phi_\pi(x)$ as

$$\langle D^+\pi^- |(\bar{c}b)_{\text{V}-\text{A}}(\bar{d}u)_{\text{V}-\text{A}}|\bar{B}^0\rangle = \langle D^+ |(\bar{c}_v b_{v'})_{\text{V}-\text{A}}|\bar{B}^0\rangle$$
$$\times \int_0^1 dx\ T(x, m_c/m_b, \mu)\phi_\pi(x, \mu).s \tag{4}$$

where $c_v, b_{v'}$ are effective fields for heavy quarks in HQET. They point out the above factorization formulae is the leading order result in Λ_{QCD}/m_b. One loop calculate is done and the result show α_s correction to leading contribution is small. However, they don't give proof of the factorization.

2.2 The large energy effective theory

The success of HQET motivates theorists to use effective field theory into wider range of application. Dugan and Grinstein had a new idea to establish a foundation for factorization on the effective field theory. They use the effective field theory to replace the intuitive "color transparency argument". In [9], Dugan and Grinstein proposed a large energy effective theory (LEET) to describe the interaction of the energetic collinear quark with soft gluons. They consider one kinematic case that the energy of the collinear quark is much lager than the momentum of soft gluon, i.e., $E \gg \Lambda_{\mathrm{QCD}}$. The central idea in LEET is that the energy of the collinear quark is unchanged which is analogous to the velocity superselection rule in HQET. The LEET is very similar to the HQET. The LEET Lagrangian is

$$\mathcal{L}_{LEET} = \psi^\dagger i n \cdot D \psi. \tag{5}$$

where n is a light-like vector which the direction is along the motion of the collinear quark and the collinear field ψ satisfies $\not{n}\psi = 0$.

If we choose the light-cone gauge $n \cdot A = 0$, the soft gluons decouple from the collinear and factorization is a trivial result. From this point of view, the color transparency is explained by that only the longitudinal gluons couple to the collinear quark and thus decouple. Although the proof of factorization in this way is too simple to be correct, the LEET is very impressive. It provide a new view of factorization. In pQCD, the diagrammatic analysis [1] is the most familiar method to prove factorization. The LEET provide an operator description and the result is automatical to all orders. The proposal of an effective Lagrangian permits us to use gauge symmetry at the Lagrangian level. The proof of factorization can be easily done in a gauge invariant way, i.e., we need not have to choose the light-cone gauge $n \cdot A = 0$. As we will show, the LEET is one part of the soft-collinear effective theory. The LEET Lagrangian given in Eq. (5) is just the leading order result.

The biggest problem of the LEET is that it cannot reproduce the long-distance physics of QCD. The reason is simple because it misses the collinear gluon degrees of freedom. Without collinear gluon, the collinear quark pair $\bar{u}d$ can not form the energetic bound state π^-. The neglect of collinear gluon is pointed out in [10]. Aglietti and Corbó point out one problem of LEET and modify the LEET by including the transverse degrees of freedom [10]. The improved LEET Lagrangian is given by

$$\mathcal{L} = \psi^\dagger \left(i n \cdot D + \frac{D_T^2}{2E} \right) \psi. \tag{6}$$

where E is the energy of the collinear quark. In this modified version, the collinear gluon is still missing. The effective theory which includes the collinear gluon is more complicated than Eq. (6) because the energy E is not conserved. Except the missing of the collinear gluon degrees of freedom, both the LEET and its modified version have the problem that there is no systematic power counting to support them. They have to wait for the next step development.

3 The QCD factorization approach and the soft-collinear effective theory

The application of the pQCD method in [2] into the exclusive B decays had been explored by many theorists. There are several different perturbative approaches appeared in the literatures. Due to the scope of this review, we focus our discussions on the recently proposed QCD factorization approach in [11, 12].

3.1 The QCD factorization approach

Beneke, Buchalla, Neubert and Sachrajda want to establish a rigorous framework for the exclusive nonleptonic B decays. The basic idea of the QCD factorization approach is that in the heavy quark limit the naive factorization is the lowest order approximation and the corrections to the naive factorization can be formulated as a factorization form up to corrections of order $\Lambda_{\mathrm{QCD}}/m_b$. The heavy quark limit $m_b \gg \Lambda_{\mathrm{QCD}}$ is the kinematic foundation of the QCD factorization approach. For $\bar{B}^0 \to \pi\pi$ decay, the factorization formula is

$$\langle \pi\pi | Q_i | \bar{B}^0 \rangle = F^{B\pi}(0) \int_0^1 dx \ T_i^I(x) \phi_\pi(x)$$
$$+ \int_0^1 d\xi dx dy \ T_i^{II}(\xi, x, y) \phi_B(\xi) \phi_\pi(x) \phi_\pi(y). \tag{7}$$

where $F^{B\pi}$ is a $B \to \pi$ form factor at $q^2 = 0$, $\phi_{\pi(B)}$ are light-cone distribution amplitudes of the pion and B meson. The $T_i^{I(II)}$ are perturbatively calculable hard scattering kernels. Compared to Eq. (4), the difference lies in the second term due to the hard spectator correction.

Ref. [12] can be considered as a systematic introduction of the pQCD method into B physics for the first time. In [12], a power counting is used to argue the validity of the QCD factorization approach. This power counting largely uses the endpoint behavior of the distribution amplitudes of mesons. Annihilation diagrams and higher Fock states of the mesons are proved to be power suppressed. The hard spectator

interaction in $\bar{B}^0 \to D^+\pi^-$ decay is suppressed if one assumes c quark is heavy.

The validity of factorization can not be based on the intuitive arguments only. It should be proved to all orders that all the soft and collinear divergences cancel or can be absorbed into the universal non-perturbative functions and the hard scattering kernels are infrared insensitive. For $B \to \pi\pi$ decays, the factorization is proved to hold in α_s order. But it is not sufficient to guarantee the validity of factorization because all the soft and collinear divergences cancel is not general. The general case is that the infrared divergences may not cancel but they can be separated out and absorbed into the definition of the non-perturbative functions. Up to now, the factorization beyond α_s order in $B \to \pi\pi$ decays has not been truly proved.

In [12], the factorization proof for $B \to D\pi$ decays at two-loop order is given. Two-loop order is equivalent to α_s^2 order because the hard spectator interaction is power suppressed for the heavy-light final states. The authors consider 62 "non-factorizable" diagrams at two loop order. Maybe their two loop order proof of factorization is the most detailed analysis in the literatures up to my knowledge. The eikonal approximation and the Ward identity are used implicitly. At two-loop order, the infrared divergences in the soft-soft, soft-collinear and collinear-collinear momentum regions cancel. The hard-collinear and hard-soft contributions contain non-cancelling infrared divergences. They can be factoriaed out and absorbed into the definition of distribution amplitude and form factor respectively. The proof of factorization in the diagrammatic analysis is intuitive, but it is impossible to go to all orders. A factorization proof of all orders is necessary.

3.2 The soft-collinear effective theory

From the LEET, we know that the effective field theory can simplify the analysis of the infrared physics and the factorization in it is automatic to all orders. One natural idea is: can we construct an effective field theory for the soft and collinear particles which reproduce all the infrared physics of QCD and can simplify the factorization proof?

Bauer el al. aim at developing a soft-collinear effective theory by generalizing the idea of LEET. They start from the study of summing Sudakov double logarithms in inclusive $B \to X_s\gamma$ decay. For $B \to X_s\gamma$ decay near the endpoint of the photon spectrum, it contains energetic light particle. The Sudakov double logarithms will appear in one loop order due to the non-cancellation of the soft and collinear divergences. The large double logarithms make the perturbation expansion ill-behaved and need to be resumed to all

orders. The Sudakov resummation is usually considered as an important but difficult part in the conventional pQCD method. By matching the full theory to a new effective theory, the large logarithms cancel and the Sudakov doule logarithms are summed by using the renormalization group equations [13]. One important thing is that Sudakov resummation in the effective theory is simpler than that in the full theory.

The analysis in [13] also shows another important thing that the effective field theory can be used in the case where the Wilson's short-distance operator product expansion (OPE) is not applicable. The idea is matching the full theory onto the effective theory where the effective operators provide the long-distance information of QCD. To this externt, the effective field theory develops the idea of OPE.

The formalism of the soft-collinear effective theory is provide in [14]. At this stage, only the ultrasoft gluons are included. The lowest order effective Lagrangian is written by

$$\mathcal{L}_{eff} = \bar{\xi}_{n_-,p'} \left[in_- \cdot D + i\not{D}_{c\perp} \frac{1}{in_+ \cdot D_c} i\not{D}_{c\perp} \right] \quad (8)$$
$$\times \frac{\not{n}_+}{2} \xi_{n_-,p} .$$

where $in_- \cdot D = in_- \cdot \partial + gn_- \cdot (A_c + A_{us})$, $in_+ \cdot D_c = \bar{\mathcal{P}} + gn_+ \cdot A_c$, $i\not{D}_{c\perp} = \bar{\mathcal{P}}_\perp + gA_{c\perp}$ and n_-, n_+ are two light-like vectors. Because the formulation is given in a hybrid position-momentum space. A momentum label operator $\mathcal{P}$ has to be introduced because the large momentum is not conserved. Beneke, Chapovsky, Diehl and Feldmann developed a position space formalism in order to avoid the complicate momentum label operator in [17].

In [18], I propose a soft-collinear effective theory which includes the soft gluons in the position space. The final SCET Languagian is

$$\mathcal{L}_{SCET} = \bar{\xi} \left[in_- \cdot D + i\not{D}_\perp \frac{1}{in_+ \cdot D} i\not{D}_\perp \right] \frac{\not{n}_+}{2} \xi. \quad (9)$$

where the covariant derivative is defined by $D = \partial - igA_c - igA_{(u)s}$.

From the above effective Lagrangian, it is easy to obtain the (ultra)soft and collinear Wilson lines. The SCET has rich symmetry structures. The Lorentz and gauge invariance are interesting in SCET. There is a new symmetry, scale symmetry. This symmetry provide an interpretation of the Bjorken's scaling and the scaling law in high energy scattering.

The application of SCET to prove the factorization in $B \to D\pi$ decays is given in [15, 16]. I give a

factorization proof in DIS in [18] and SCET can also be applied into multi-body B decays, such as $B \rightarrow DKK$ decays [19].

4 The questions about the QCD factorization approach

Although SCET provides a new theoretical framework, the practical calculations of exclusive B decays still rely on some factorization formulae, such as the QCD factorization approach. By use of this opportunity, I want to express my personal opinions on the QCD factorization approach. I will ask some conceptual questions about it.

I. Is the factorization in $B \rightarrow \pi\pi$ decays proved?

As we have discussed above, the factorization in $B \rightarrow \pi\pi$ decays is given only at α_s order. This is not sufficient for the validity of factorization. We can say that the QCD factorization approach has not been proved as a "theory". Even for $B \rightarrow D\pi$ decays, the factorization are based on some kinematical assumptions, such as $m_b \sim m_c \rightarrow \infty$ or the ratio m_b/m_c is fixed. The real world is: $m_b \approx 4.5\text{GeV}$, $m_c \approx 1.5\text{GeV} \sim \sqrt{m_b\Lambda_{\text{QCD}}}$. Although one can assume a limit case for the theoretical purpose, the relation between the ideal world and the real world needs to be explored.

II. Is the QCD factorization approach correct in the $m_b \rightarrow \infty$ limit?

This question is related to the first question. This time we will not concern the technical complications about the factorization proof but the general argument of factorization. The argument is Bjorken's "color transparency". One strange thing for me is why the color transparency can be applied for one pion but not for another pion in $B \rightarrow \pi\pi$ decays. The two pions are both energetic and have the same momentum in the rest frame of B meson, but they are treated unequally. The factorization formula in Eq. (7) contains two terms, one is related to the naive factorization approach, the other likes the hard scattering approach. The two different terms make the factorization formula un-natural. One associated problem is: the renormalization scale μ in first term is at order of m_b, while in the next term $\mu \sim \sqrt{m_b\Lambda_{\text{QCD}}}$. There are two large scales in $B \rightarrow \pi\pi$ decays. So it is a multi-scales problem. My oppinion is that we should be more serious about his problem.

III. Is $B \rightarrow \pi$ form factor baisc?

This is a very controversial topic. In the QCD factorization approach, it is assumed as a basic function. But this function is different from other fundamental non-perturbative QCD fucntions such as the Isgur-Wise function, pion distribution amplitudes etc. $B \rightarrow \pi$ form factor is not and can not be represented as a dimensionless function. From the soft-collinear effective theory which is scale invariant, the basic non-perturbative functions should be dimensionaless except the dependence on the renormalization μ. The $B \rightarrow \pi$ form factor does not satisfy this criterion. In fact, $B \rightarrow \pi$ form factor contains very complicate QCD dynamics which includes scales of m_b, $\sqrt{m_b\Lambda_{\text{QCD}}}$ and scales between $\sqrt{m_b\Lambda_{\text{QCD}}}$ and Λ_{QCD}. From this point of view, the QCD factorization seems more like a phenomenological "approach" rather than a "theory".

5 Conclusions

We have reviewed the theoretical developments from the naive factorization approach to the soft-collinear effective theory. In order to understand the factorization in exclusive B decays, new ideas and new theories or approaches are produced. The soft-collinear effective theory is a rigorous theory. It provides a theoretical foundation of the factorization theorem in pQCD and a unified framework to study the inclusive and exclusive hard QCD processes. The soft-collinear effective theory is the second theoretical contribution of B physics to QCD. The first is HQET.

The QCD factorization approach can not solve the full complications of exclusive B decays. The application of the soft-collinear effective in B decays is still limited because the momentum of quarks in hadrons depend on the non-perturbative dynamics for exclusive processes. The final solution of B decays must rely on the solution on the quark confinement and non-perturbative problems. If these fundamental problems of the strong interaction were solved by the theorists in B physics, it will not be a miracle. This is the great challenge of B physics.

Acknowledgments

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